STATISTICAL PROCESS CONTROL
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- Application of statistical techniques to
  - The control of processes
  - Ensure that processes meet standards

SPC is a process used to monitor standards by taking measurements and corrective action as a product or service is being produced.
STATISTICAL PROCESS CONTROL

• All processes are subject to a certain degree of variability.

• Types of variability:
  • Natural or common causes
  • Special or assignable causes
STATISTICAL PROCESS CONTROL

- SPC → to measure performance of a process
- A process is said to be operating *in statistical control* when the only source of variation is common (natural) causes.
- Provides a statistical signal when assignable causes are present
- Detect and eliminate assignable causes of variation
Natural Variations

- Also called common causes
- Affect virtually all production processes
- Expected amount of variation
- Output measures follow a probability distribution
- For any distribution there is a measure of central tendency and dispersion
- If the distribution of outputs falls within acceptable limits, the process is said to be “in control”
ASSIGNABLE VARIATIONS

- Also called special causes of variation
  - Generally this is some change in the process
- Variations that can be traced to a specific reason
- The objective is to discover when assignable causes are present
  - Eliminate the bad causes
  - Incorporate the good causes
To measure the process, we take samples and analyze the sample statistics following these steps

(a) Samples of the product, say five boxes of cereal taken off the filling machine line, vary from each other in weight

Each of these represents one sample of five boxes of cereal
To measure the process, we take samples and analyze the sample statistics following these steps.

(b) After enough samples are taken from a stable process, they form a pattern called a distribution.

Figure S6.1
SAMPLES

To measure the process, we take samples and analyze the sample statistics following these steps.

(c) There are many types of distributions, including the normal (bell-shaped) distribution, but distributions do differ in terms of central tendency (mean), standard deviation or variance, and shape.
To measure the process, we take samples and analyze the sample statistics following these steps:

(d) If only natural causes of variation are present, the output of a process forms a distribution that is stable over time and is predictable.
SAMPLES

To measure the process, we take samples and analyze the sample statistics following these steps

(e) If assignable causes are present, the process output is not stable over time and is not predictable

Figure S6.1
CONTROL CHARTS

Constructed from historical data, the purpose of control charts is to help **distinguish** between **natural variations** and variations due to **assignable causes**
PROCESS CONTROL

(a) In statistical control and capable of producing within control limits

(b) In statistical control but not capable of producing within control limits

(c) Out of control

Frequency

Lower control limit

Upper control limit

Size

(weight, length, speed, etc.)
CENTRAL LIMIT THEOREM

Regardless of the distribution of the population, the distribution of sample means drawn from the population will tend to follow a normal curve

1. The mean of the sampling distribution ($\bar{x}$) will be the same as the population mean $\mu$

$$\bar{x} = \mu$$

2. The standard deviation of the sampling distribution ($\sigma_{\bar{x}}$) will equal the population standard deviation ($\sigma$) divided by the square root of the sample size, $n$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$
POPCULATION AND SAMPLING DISTRIBUTIONS

Three population distributions

- Beta
- Normal
- Uniform

Distribution of sample means

Mean of sample means $= \bar{x}$

Standard deviation of the sample means $= \sigma_x = \frac{\sigma}{\sqrt{n}}$

$95.45\%$ fall within $\pm 2\sigma_x$

$99.73\%$ of all $\bar{x}$ fall within $\pm 3\sigma_x$
SAMPLING DISTRIBUTION

\[ x = \mu \]  
(mean)
## TYPES OF DATA

<table>
<thead>
<tr>
<th>Variables</th>
<th>Attributes</th>
</tr>
</thead>
<tbody>
<tr>
<td>✔ Characteristics that can take any real value</td>
<td>✔ Defect-related characteristics</td>
</tr>
<tr>
<td>✔ May be in whole or in fractional numbers</td>
<td>✔ Classify products as either good or bad or count defects</td>
</tr>
<tr>
<td>✔ Continuous random variables</td>
<td>✔ Categorical or discrete random variables</td>
</tr>
</tbody>
</table>
CONTROL CHARTS FOR VARIABLES

- For variables that have continuous dimensions
  - Weight, speed, length, strength, etc.
- $\bar{x}$-charts are to control the central tendency of the process
- $R$-charts are to control the dispersion of the process
- These two charts must be used together
SETTING CHART LIMITS

For $\bar{x}$-Charts when we know $\sigma$

Upper Control limit (UCL) $= \bar{x} + z\sigma_{\bar{x}}$

Lower Control limit (LCL) $= \bar{x} - z\sigma_{\bar{x}}$

where

$\bar{x}$ = mean of the sample means or a target value set for the process

$z$ = number of normal standard deviations

$\sigma_{\bar{x}}$ = standard deviation of the sample means

$= \frac{\sigma}{\sqrt{n}}$

$\sigma$ = population standard deviation

$n$ = sample size
EXAMPLE 1

- The weights of Oat Flakes within a large production lot are sampled each hour. Managers want to set control limits that include 99.73% of the sample means.
- Randomly select and weight 9 boxes each hour.
- Population standard deviation ($\sigma$) is known to be 1 ounce.
- Each of the boxes randomly selected in hours 1 through 12.
### EXAMPLE 1: SETTING CONTROL LIMITS

#### Hour 1

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Weight of Oat Flakes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>18</td>
</tr>
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<td>5</td>
<td>17</td>
</tr>
<tr>
<td>6</td>
<td>16</td>
</tr>
<tr>
<td>7</td>
<td>15</td>
</tr>
<tr>
<td>8</td>
<td>17</td>
</tr>
<tr>
<td>9</td>
<td>16</td>
</tr>
</tbody>
</table>

**n = 9**

**Mean** = 16.1,  
**\( \sigma = 1 \)**

#### Hour Mean

<table>
<thead>
<tr>
<th>Hour</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16.1</td>
</tr>
<tr>
<td>2</td>
<td>16.8</td>
</tr>
<tr>
<td>3</td>
<td>15.5</td>
</tr>
<tr>
<td>4</td>
<td>16.5</td>
</tr>
<tr>
<td>5</td>
<td>16.5</td>
</tr>
<tr>
<td>6</td>
<td>16.4</td>
</tr>
<tr>
<td>7</td>
<td>15.2</td>
</tr>
<tr>
<td>8</td>
<td>16.4</td>
</tr>
<tr>
<td>9</td>
<td>16.3</td>
</tr>
</tbody>
</table>

#### Hour Mean

<table>
<thead>
<tr>
<th>Hour</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>15.2</td>
</tr>
<tr>
<td>8</td>
<td>16.4</td>
</tr>
<tr>
<td>9</td>
<td>16.3</td>
</tr>
<tr>
<td>10</td>
<td>14.8</td>
</tr>
<tr>
<td>11</td>
<td>14.2</td>
</tr>
<tr>
<td>12</td>
<td>17.3</td>
</tr>
</tbody>
</table>

**For 99.73% control limits, **\( z = 3 \)**

**UCL** = \( \bar{x} + z\sigma_{\bar{x}} = 16 + 3 \left( \frac{1}{\sqrt{9}} \right) = 17 \) ozs

**LCL** = \( \bar{x} - z\sigma_{\bar{x}} = 16 - 3 \left( \frac{1}{\sqrt{9}} \right) = 15 \) ozs
EXAMPLE 1: SETTING CONTROL LIMITS

- The means of recent sample averages falls outside the UCL and LCL of 17 and 15.
- The process is NOT in control
SETTING CHART LIMITS

For R-Charts

Upper Control limit (UCL\(_R\)) = D_4 \bar{R}

Lower Control limit (LCL\(_R\)) = D_3 \bar{R}

where

\[ \bar{R} = \text{average range of the samples} \]

D_3 and D_4 = control chart factors from Table
<table>
<thead>
<tr>
<th>Sample Size</th>
<th>Mean Factor</th>
<th>Upper Range</th>
<th>Lower Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>$A_2$</td>
<td>$D_4$</td>
<td>$D_3$</td>
</tr>
<tr>
<td>2</td>
<td>1.880</td>
<td>3.268</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1.023</td>
<td>2.574</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>.729</td>
<td>2.282</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>.577</td>
<td>2.115</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>.483</td>
<td>2.004</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>.419</td>
<td>1.924</td>
<td>0.076</td>
</tr>
<tr>
<td>8</td>
<td>.373</td>
<td>1.864</td>
<td>0.136</td>
</tr>
<tr>
<td>9</td>
<td>.337</td>
<td>1.816</td>
<td>0.184</td>
</tr>
<tr>
<td>10</td>
<td>.308</td>
<td>1.777</td>
<td>0.223</td>
</tr>
<tr>
<td>12</td>
<td>.266</td>
<td>1.716</td>
<td>0.284</td>
</tr>
</tbody>
</table>
EXAMPLE 2

- The average range of a product at Clinton manufacturing is 5.3 pounds. With a sample size of 5, the owner wants to determine the UCL and LCL.

- From Table of Control Chart Factor
  - \( n = 5 \)
  - \( D_4 = 2.115 \)
  - \( D_3 = 0 \)
EXAMPLE 2: SETTING CONTROL LIMITS

\[
UCL_R = D_4 \bar{R} \\
= (2.115)(5.3) \\
= 11.2 \text{ pounds}
\]

\[
LCL_R = D_3 \bar{R} \\
= (0)(5.3) \\
= 0 \text{ pounds}
\]

\begin{align*}
\text{UCL} &= 11.2 \\
\text{Mean} &= 5.3 \\
\text{LCL} &= 0
\end{align*}
MEAN AND RANGE CHARTS

(a) These sampling distributions result in the charts below

\( \bar{x} \)-chart
- (Sampling mean is shifting upward but range is consistent)
- (\( \bar{x} \)-chart detects shift in central tendency)

R-chart
- (R-chart does not detect change in mean)

UCL
LCL
MEAN AND RANGE CHARTS

(b) These sampling distributions result in the charts below:

(x-chart) does not detect the increase in dispersion.

(R-chart) detects increase in dispersion.

(Sampling mean is constant but dispersion is increasing.)
CONTROL CHARTS FOR ATTRIBUTES

- For variables that are categorical
  - Good/bad, yes/no, acceptable/unacceptable
- Measurement is typically counting defectives
- Charts may measure
  - Percent defective (p-chart)
  - Number of defects (c-chart)
CONTROL LIMITS FOR P-CHARTS

Population will be a binomial distribution, but applying the Central Limit Theorem allows us to assume a normal distribution for the sample statistics.

\[
UCL_p = \bar{p} + z\sigma_{\hat{p}} \\
LCL_p = \bar{p} - z\sigma_{\hat{p}}
\]

where

- \(\bar{p}\) = mean fraction defective in the sample
- \(z\) = number of standard deviations
- \(\sigma_{\hat{p}}\) = standard deviation of the sampling distribution
- \(n\) = sample size

\[
\sigma_{\hat{p}} = \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}
\]
EXAMPLE 3

• Clerk at IIMS key in thousands of insurance records each day for a variety of client firms. The CEO wants to set control limits to include 99.73% of the random variation in data entry process when it is in control.

• Sample of work for 20 clerks are gathered.

• 100 records entered by each clerk are examine and the number of errors are counted.

• The fraction defective of each sample are computed.
**EXAMPLE 3: P-CHART FOR DATA ENTRY**

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Number of Errors</th>
<th>Fraction Defective</th>
<th>Sample Number</th>
<th>Number of Errors</th>
<th>Fraction Defective</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>.06</td>
<td>11</td>
<td>6</td>
<td>.06</td>
</tr>
<tr>
<td>2</td>
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<tr>
<td>5</td>
<td>4</td>
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</tr>
<tr>
<td>9</td>
<td>3</td>
<td>.03</td>
<td>19</td>
<td>0</td>
<td>.00</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>.02</td>
<td>20</td>
<td>4</td>
<td>.04</td>
</tr>
</tbody>
</table>

Total = 80
EXAMPLE 3: SETTING CONTROL LIMITS

\[ \bar{p} = \frac{\text{total number of errors}}{\text{total number of records examined}} = \frac{80}{(100)(20)} = 0.04 \]

\[ \sigma_{\bar{p}} = \sqrt{\frac{(0.04)(1 - 0.04)}{100}} = 0.02 \]

\[ \text{UCL}_p = \bar{p} + z\sigma_{\bar{p}} = 0.04 + 3(0.02) = 0.1 \]

\[ \text{LCL}_p = \bar{p} - z\sigma_{\bar{p}} = 0.04 - 3(0.02) = 0 \]

We cannot have a negative percent defective
EXAMPLE 3: P-CHART FOR DATA ENTRY

Possible assignable causes present

• One data-entry clerk (no 17) is out of control.
• Need to examine that individual’s work a bit more closely to see if a serious problem exists.
EXAMPLE 3: **P-CHART FOR DATA ENTRY**

- Two data-entry clerk (no 3 & 19) is reported no errors.
- Need to examine that individual’s work if there are skills or processes that can be applied to other operators.
CONTROL LIMITS FOR C-CHARTS

Population will be a Poisson distribution, but applying the Central Limit Theorem allows us to assume a normal distribution for the sample statistics

\[
\text{UCL}_c = \bar{c} + 3\sqrt{\bar{c}} \\
\text{LCL}_c = \bar{c} - 3\sqrt{\bar{c}}
\]

where \( \bar{c} = \text{mean number defective in the sample} \)
EXAMPLE 4

- TRC company receives several complaints per day about the behavior of its drivers. Over a 9-day period (days are the units of measure), the owner, received the following numbers of calls from irate passengers:
  \[3, 0, 8, 9, 6, 7, 4, 9, 8\]

- Total complaints = 54

- The owner wants to compute 99.73% control limits
EXAMPLE 4: \textbf{C-CHART}

\[ \bar{c} = 54 \text{ complaints/9 days} = 6 \text{ complaints/day} \]

\[
\text{UCL}_c = \bar{c} + 3\sqrt{c} \\
= 6 + 3\sqrt{6} \\
= 13.35
\]

\[
\text{LCL}_c = \bar{c} - 3\sqrt{c} \\
= 6 - 3\sqrt{6} \\
= 0
\]
PATTERNS IN CONTROL CHARTS

Normal behavior. Process is “in control.”
One plot out above (or below). Investigate for cause. Process is “out of control.”
PATTERNS IN CONTROL CHARTS

Upper control limit

Target

Lower control limit

Trends in either direction, 5 plots. Investigate for cause of progressive change.
PATTERNS IN CONTROL CHARTS

Upper control limit

Target

Lower control limit

Two plots very near lower (or upper) control. Investigate for cause.
Run of 5 above (or below) central line. Investigate for cause.
Erratic behavior. Investigate.