Business Decision techniques (BUSS0204)

Probability

Probability is the chance that something will happen. It is a measure of likelihood and can be stated as a percentage, a ratio, or more usually as a number from 0 to 1.

Consider the following.

- Probability = 0 = impossibility
- Probability = 1 = certainty
- Probability = 1/2 = a 50% chance of something happening
- Probability = 1/4 = a 1 in 4 chance of something happening

What is probability?

- A probability is the chance that something will happen.
- Eg:
    - (a) I am 100% confidence that I will pass the exam.
    - (b) About 50% of the staff are not satisfied with their salary.
    - (c) The chance for you to win this battle is very low.

Experiment

- An **experiment** is a situation involving chance or probability that leads to results called outcomes.
- Eg:
  - 1. Rolling a fair die.
  - 2. Tossing a coin.
  - 3. Choosing a student from the class.

<u>Sample Space, (S)</u>

- is the set of all possible outcomes of an experiment.
- Eg:
  - 1. All the outcomes for the experiment ‘Rolling a fair die’: <u>1, 2, 3, 4, 5 and 6</u>.
  - 2. All the outcomes for the experiment ‘Tossing a coin’: <u>Head and Tail.</u>
  - 3. All the outcomes for the experiment ‘Choosing a student from the class’: _____________

Event

- An event is one or more outcomes that may occur.
- Eg:
  - 1. Getting even numbers for the experiment ‘Rolling a fair die’. <u>2, 4 and 6.</u>
  - 2. Getting multiplies of 3 for the experiment ‘Rolling a fair die’. <u>3 and 6.</u>
  - 3. Getting long hair students for the experiment ‘Choosing a student from the class’._______

Outcomes

- An **outcome** is the result of a single trial of an experiment.
- Eg:
  - 1. Getting a ‘1’ in a rolling of a fair die.
  - 2. Getting a ‘head’ in a tossing of a coin.

Probability

$$P(A) = \frac{n(A)}{n(S)}$$

Where:

- n(A) = the number of outcomes in event set A
- n(S) = sample space (All possible outcome)

- P(A) = 0, if the event is impossible to occur.
- P(A) = 1, if the event is certain to occur.
- If A denotes as an event 'A occurs' and denotes an event 'A does not occur',
- Then

$$P(A) + P(\bar{A}) = 1$$

Question 2.1

- What is probability of getting a number 4 with the rolling of a single die? Hence find the probability of not getting a number 4.

Question 2.2

Form	1	2	3	4	5
No. of students	40	80	70	60	50

- The table shows the number of students in different forms in a school. A student is selected from the school. Find the probability that
  - a Form 2 student is selected.
  - a Form 5 student is selected.

Question 2.3

- The test marks for mathematics in a class are recorded as shown below:

Test marks	40	50	60	70	80	90	100
No. of students	4	8	10	12	8	9	2

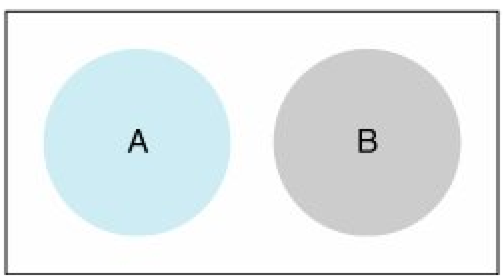
  - If a student in the class is at random, what is the probability that he or she is getting 70 marks?

  - Find the probability of a student, selected at random, scoring 90 marks and above.

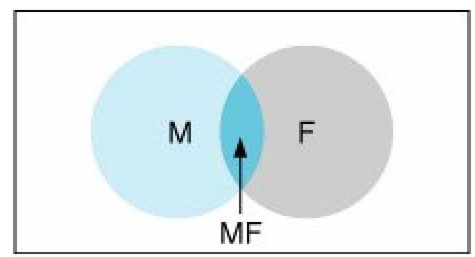
Law of probability

- The Additional Rule
- The Multiplication Rule
- Conditional probabilities and Bayes' theorem

Venn Diagrams



Mutually exclusive



Non-mutually exclusive

The Addition Rule

<u>Mutually Exclusive Events</u>

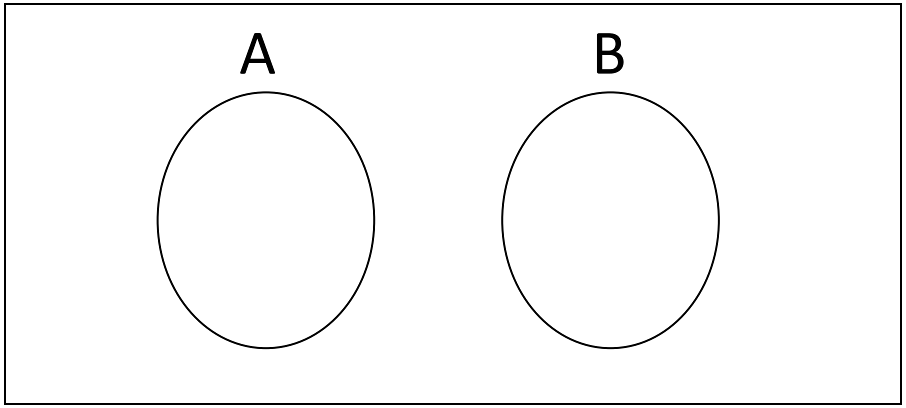
- – when two or more events cannot *occur together* simultaneously.
- Eg.

  (a) I am female and I am male. – Impossible!

  (b) I am 6 years old and I am 25 years old. – Impossible!

  (c) I like apple and I dislike apple. – Something is wrong…

Venn diagram



- The rule of addition for mutually exclusive events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = 0$$

$$P(A \cup B) = P(A) + P(B)$$

Note: $\cup$ means OR! $\cap$ means AND!

Question 2.4

- John and Owen applied for a scholarship and only **one student** will be awarded the scholarship. If the probability for John to be awarded the scholarship is 0.3 and that for Owen is 0.4, what is the probability that, either John or Owen to win the scholarship?

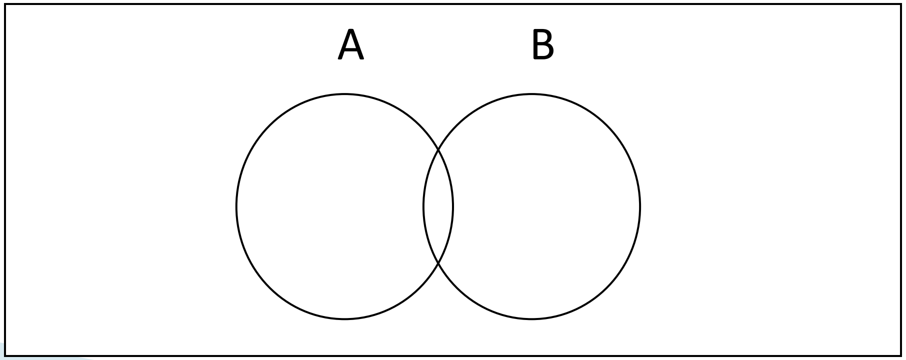
The Addition Rule

<u>Mutually non-exclusive events</u>

- – when two or more events can *occur* simultaneously
- Eg.

  Students are allowed to take either Mathematics or Statistics or both subjects this semester

Venn diagram



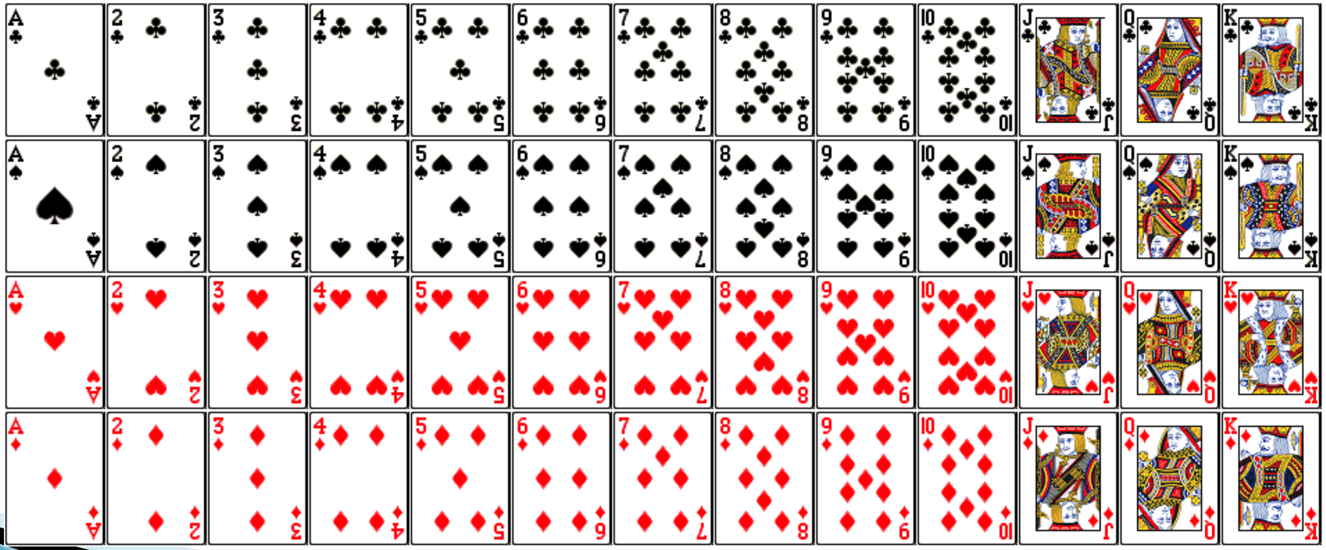
- The rule of addition for mutually non-exclusive events:

$$P(A \cap B) \neq 0$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Question 2.5

- Find the probability of drawing a single card with either a ‘7’ or a ‘heart’ from a pack of 52 playing cards.



The Multiplication Rule

Independent Events

- Occurrence one event cannot influence the occurrence of another event.

- Eg:

  Being a lecturer and having a short hair.

  →Both of these are independent on each other!

Question 2.6

- Two cards are drawn one after another from a pack of 52 playing cards. The first card is ***replaced*** before drawing the second card. What is the probability of getting a ‘king’ in the second card?

The rule of multiplication for independent events:

- If two events A and B are independent such that

$$P(A) \neq 0 \;\; and \;\; P(B) \neq 0,$$

- Then

$$P(A \cap B) = P(A) \times P(B)$$

Question 2.7

- There are 4 red pens and 5 blue pens in a box. Two pens are to be withdrawn one after another successively. However, the first pen drawn is replaced. What is the probability of drawing:

  a) the first pen is red and the second pen is blue?

  b) one red pen and one blue pen?

<u>Dependent Events</u>

- The probability of the second event ***depends*** upon the first event.
- Eg: Obtained high school qualification and obtained bachelor honour.

Question 2.8

- A bag contains 3 red balls and 4 blue balls. One ball is withdrawn, the colour noted and the ball ***is not replaced***. A second ball is then withdrawn. What is the probability that the second ball is red?

Conditional probability

- The probability that an event will occur, given that one or more other events have occurred.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Question 2.9

- A jar contains black and white marbles. Two marbles are chosen without replacement. The probability of selecting a black marble and then a white marble is 0.34, and the probability of selecting a black marble on the first draw is 0.47. What is the probability of selecting a white marble on the second draw, given that the first marble drawn was black?

Solving probability using Contingency Table

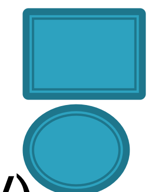
Question 2.10

- The management of a computer shop recorded the sales of 250 units of computers in one month. Of those 250 units sold, 180 units belonged to Pentium II model, of which 30 units were sold with 17 inch monitors. 25 units of Pentium I model were sold with 17 inch monitors. The rest of the units were sold with 15 inch monitors. Find the probability that a unit sold, selected at random,
  - a) has a 15 inch monitor
  - b) is a Pentium I model with a 15 inch monitor
  - c) either is a Pentium II model or has a 15 inch monitor

	15 inch monitor	17 inch monitor	
Pentium I	45	25	70
Pentium II	150	30	180
	195	55	250

Decision Trees

- Pictorial method of showing a sequence of inter-related decisions and outcomes.
- Three components:
  - Decision points
  - Outcomes
  - Outcome values (EV)

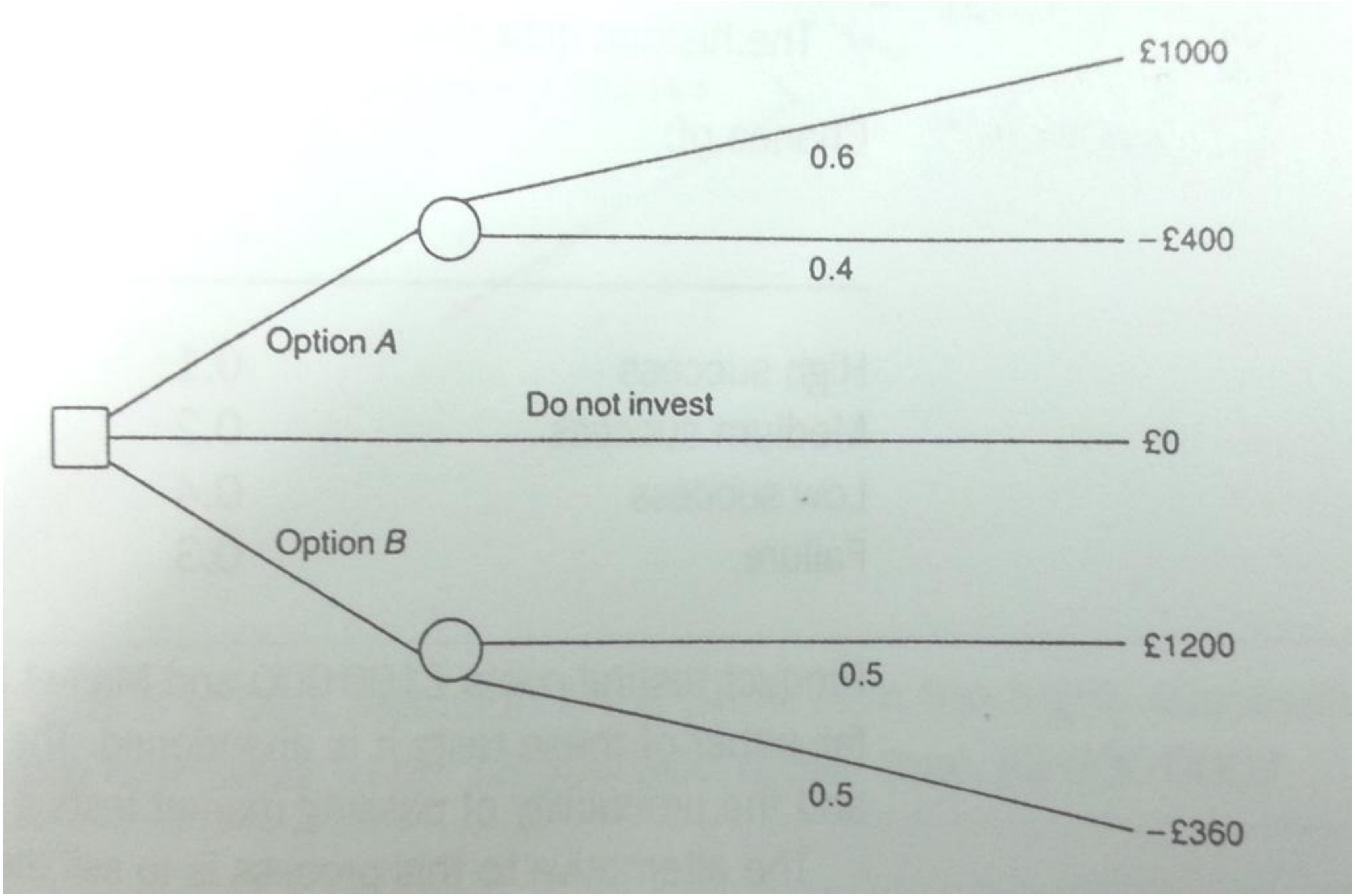


Step drawing decision tree

1) Start from left to right, should include decision nodes, outcome nodes
2) Put all the relevant information into the tree graph (outcome values, probability)
3) Calculate the EV based on outcome values and the probabilities
4) Make decisions based on the EV. The decisions with the highest EV will be selected.

Question 2.11

you are given two opportunities to invest your savings. The first opportunity, *option A*, is forecast to give a profit of \$1000 with a probability of 0.6 and a loss of \$400 with a probability of 0.4. the second opportunity, *option B*, is forecast to give a profit of \$1200 with a probability of 0.5 and a loss of \$360. these two opportunities actually give you three choices, the third choice being *not to invest*.


£1000
0.6
−£400
0.4
Option A
Do not invest
£0
Option B
£1200
0.5
0.5
−£360

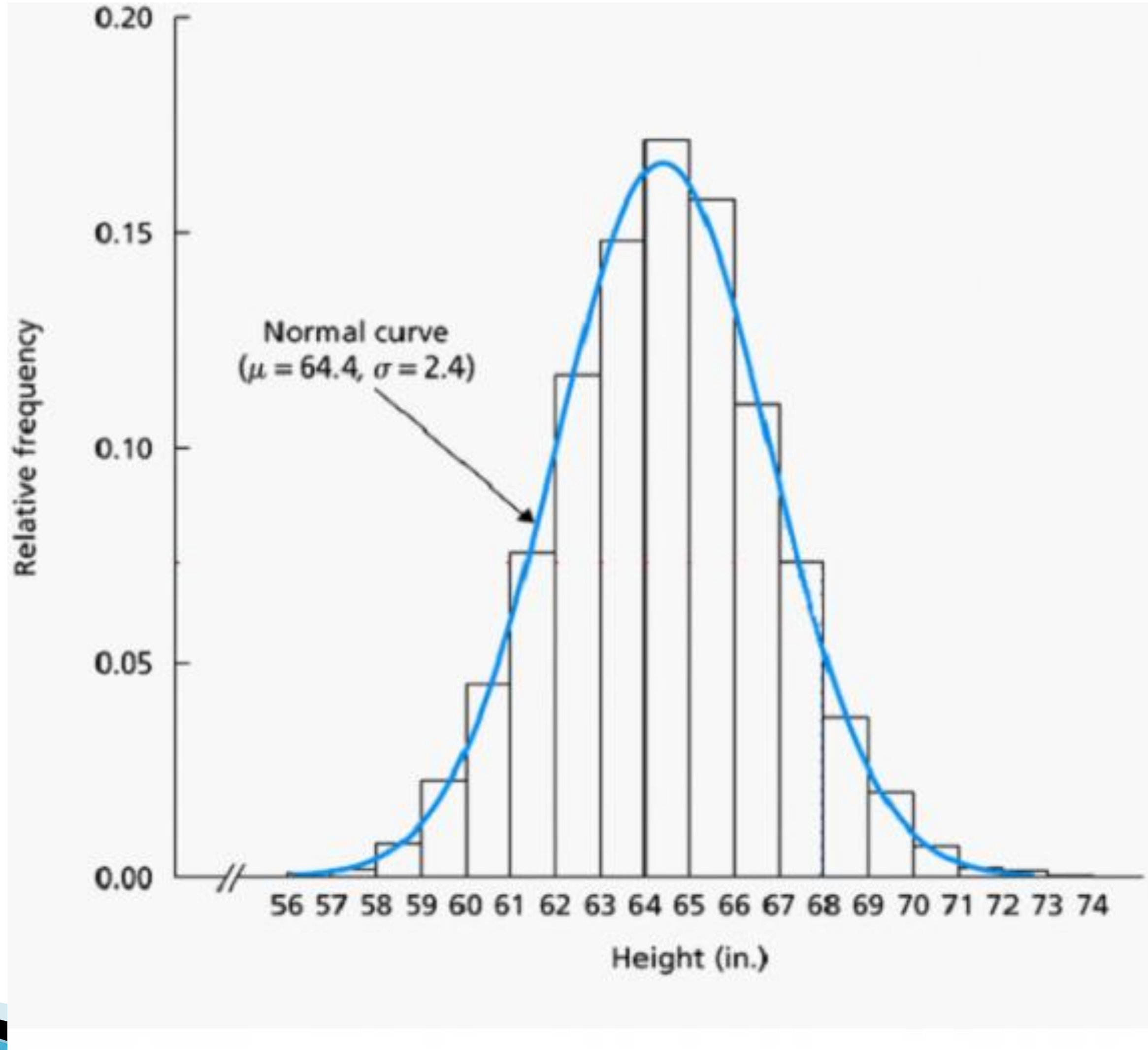
Probability Distribution

Normal Distribution

- The normal distribution is pattern for the distribution of a set of data which follows a ***bell shaped curve.***

- The bell shaped curve has several properties:
  - The curve ***concentrated in the center and decreases on either side***. This means that the data has less of a tendency to produce unusually extreme values, compared to some other distributions.
  - The bell shaped curve is ***symmetric***.

Normal Distribution



Normal Distribution

- The normal distribution is a ***continuous probability distribution***. This has several implications for probability.
- The total area under the normal curve is equal to 1.
- Find the area under the curve, means finding the probability of the variables.

How to find the probability of Normal Distribution?

- ***The standard normal distribution:*** $Z \sim N(0, 1)$
- Which having mean, $\mu = 0$ and variance, $\sigma^2 = 1$.
- ***We need standard normal table to look for probabilities.***

Z statistic

- Z tables give the area in the tails of the normal distribution
- If Z = +, then the area is in the right hand tail
- If Z = – , then the area is in the left hand tail

Question 3.1:

- If $Z \sim N(0, 1)$, find
  - (a) $P(Z > 0.5)$
  - (b) $P(Z > 1.03)$
  - (c) $P(Z < 2.1)$
  - (d) $P(Z < 0.48)$

Question 3.2:

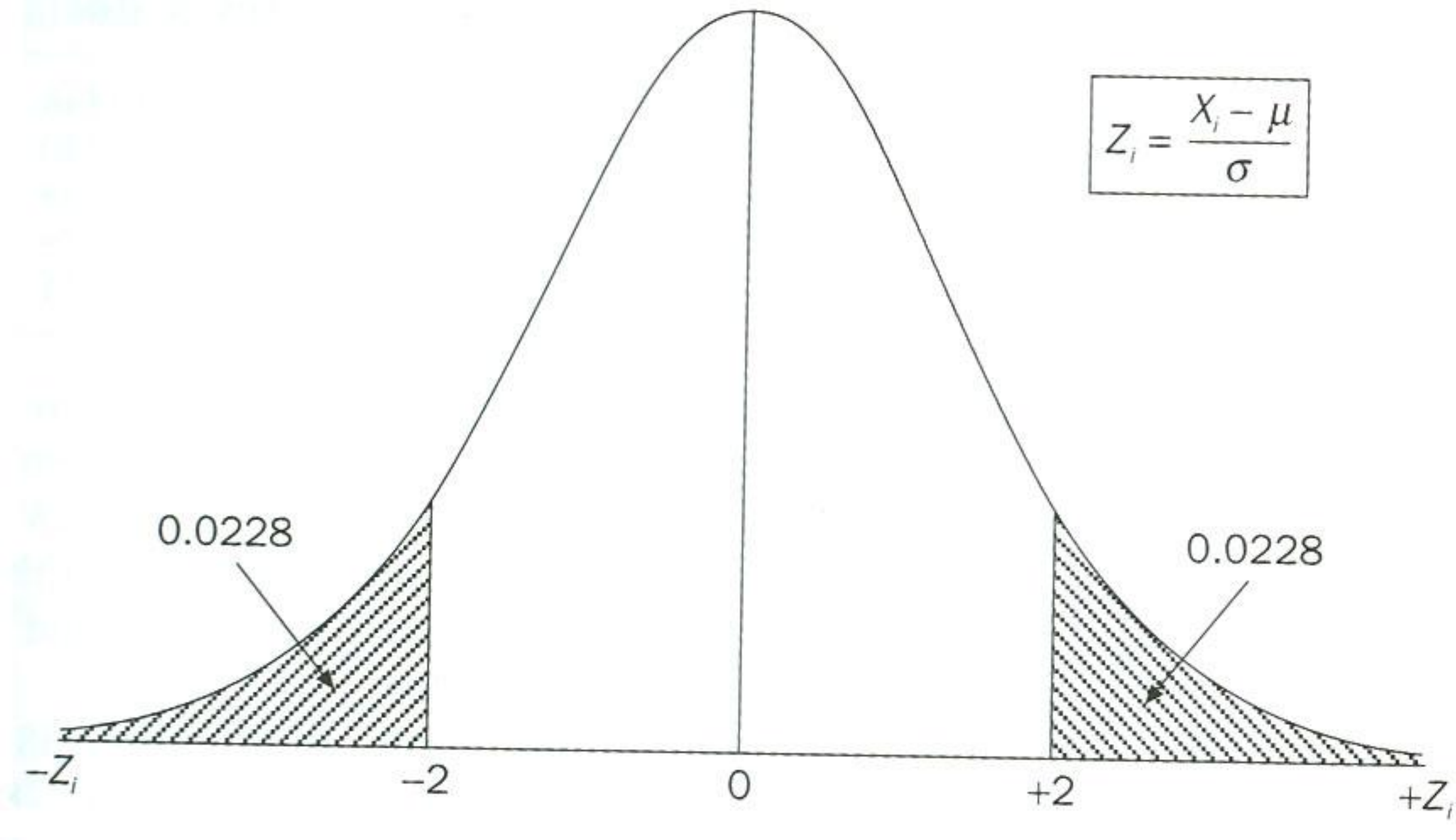
- If Z~N(0, 1), find
  - (a) $P(Z > -0.6)$
  - (b) $P(1.2 < Z < 2.4)$
  - (c) $P(-1.2 < Z < -0.8)$

Normal distribution

- **Z-statistic:** can use to establish the number of standard deviations from the mean and work out probabilities

$$Z = \frac{X_i - \mu}{\sigma}$$

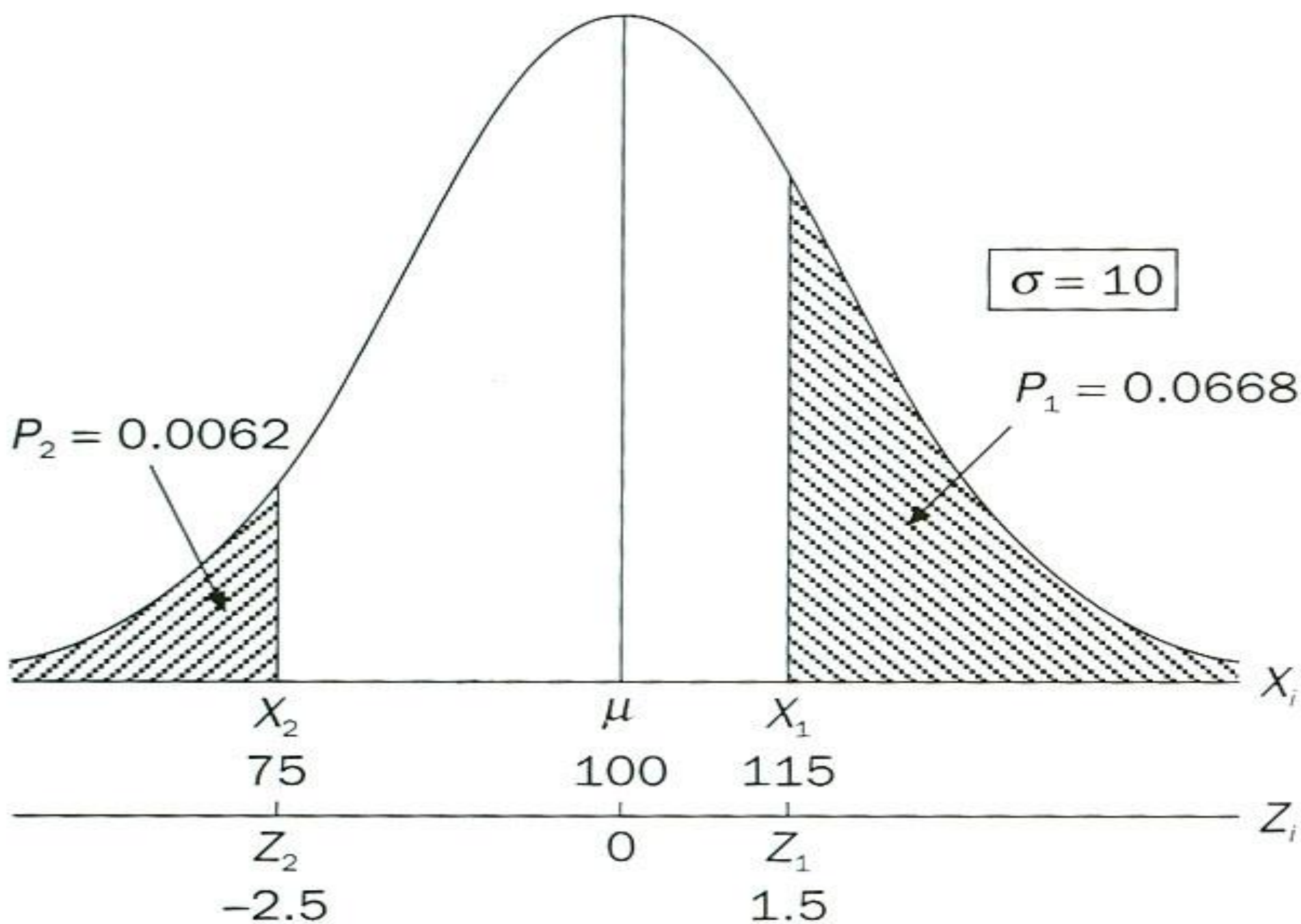
where X_i = variable value,
μ = arithmetic mean, σ = standard deviation


$Z_i = \frac{X_i - \mu}{\sigma}$
0.0228
0.0228
$-Z_i$
-2
0
$+2$
$+Z_i$

Question 3.3

- Suppose a variable has a normal distribution with a mean of 100 and a standard deviation of 10.
- Find the probability of an individual value (X_i) being:

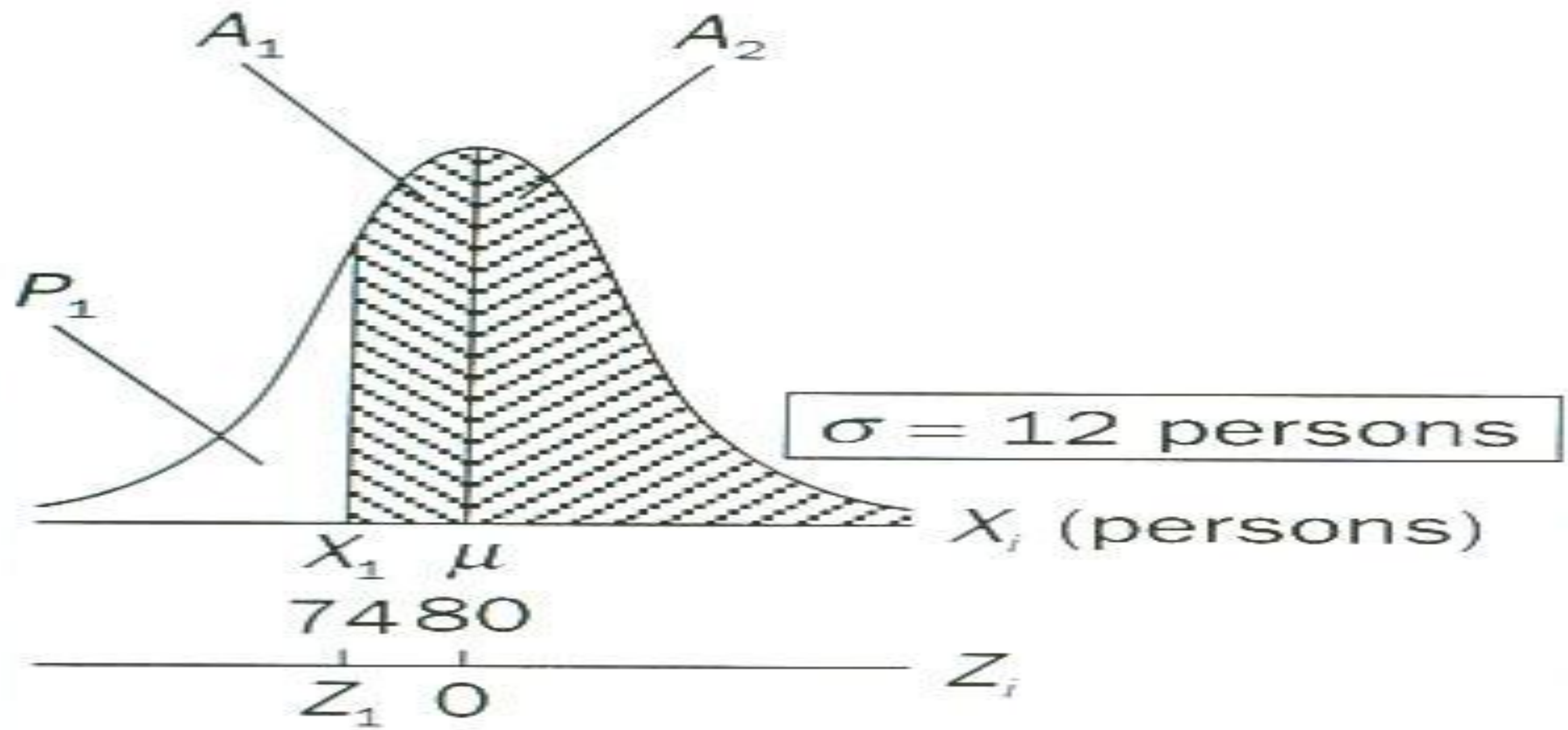
a) 115 or more
b) 75 or less
c) Between 75 and 115



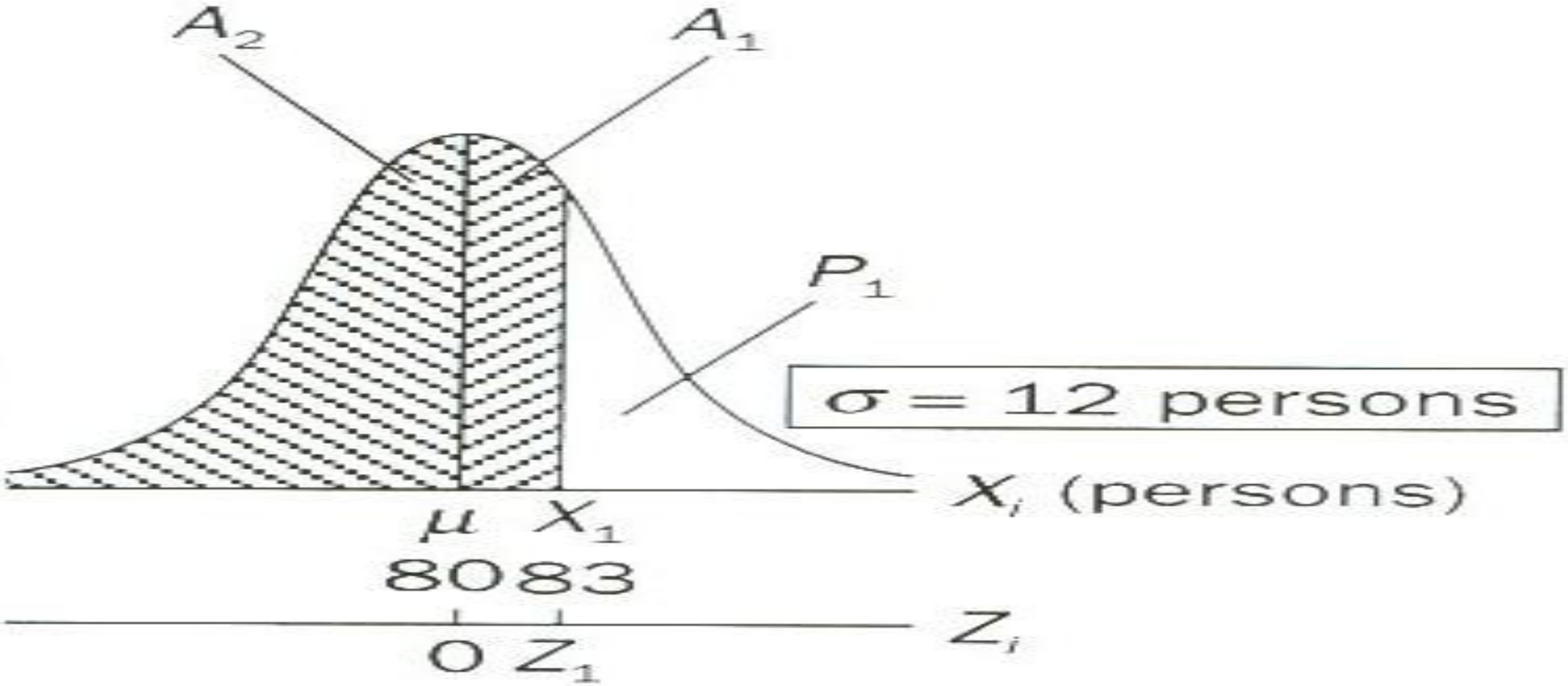
σ = 10
P₁ = 0.0668
P₂ = 0.0062
X₂
μ
X₁
Xᵢ
75
100
115
Z₂
0
Z₁
Zᵢ
–2.5
1.5

Question 3.4

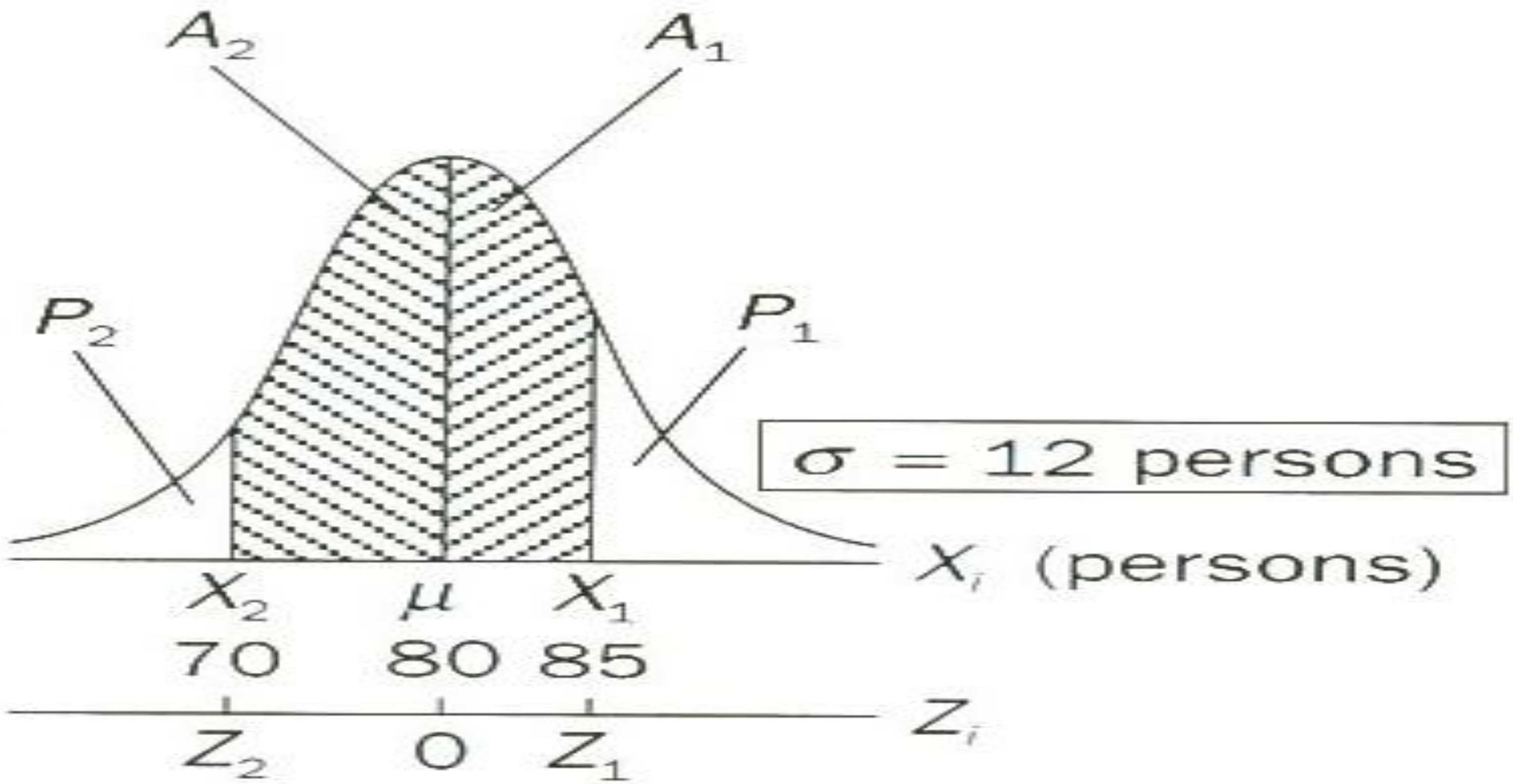
- The attendance at a night club is normally distributed, with a mean of 80 people and a standard deviation of 12 people. What is the probability that on any given night the attendance is:
  a) 74 people or more
  b) 83 people or less
  c) Between 70 and 85 people?



(a) $X_i \geq 74$ persons



(b) $X_i \le 83$ persons



(c) $70 \leq X_i \leq 85$ persons

Sampling and central limit theorem

Random sampling

- Items selected in no systematic way (‘at random’)
- All items potentially have same ‘chance’ of selection
- *Random number tables* can help here: e.g. if telephone directory is sampling frame, then such tables identify each page and line of the directory to select each person for sample
- Useful for statistical inference

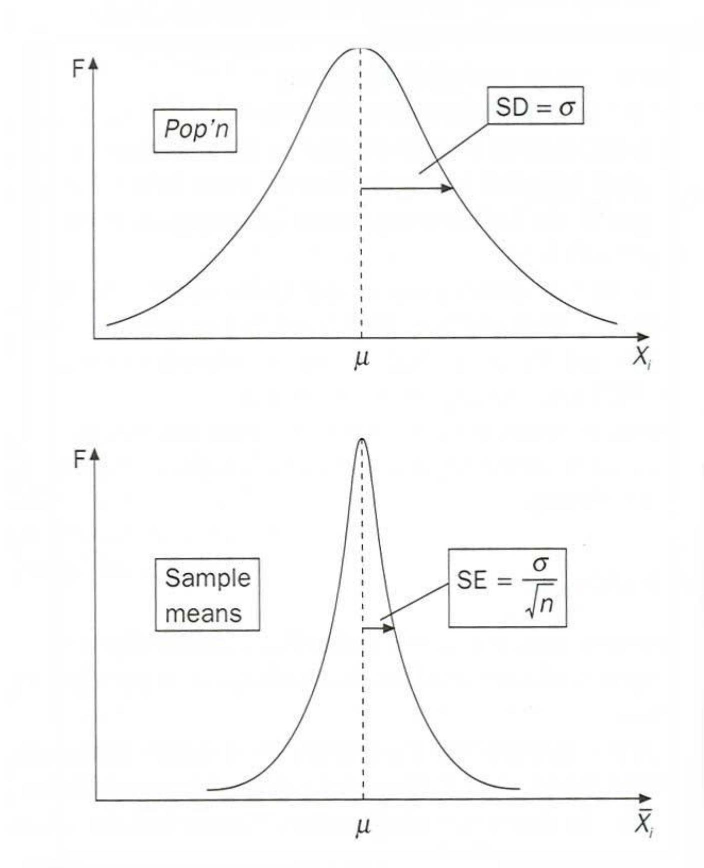
Sample size

- Sample size of 30 or more permits important statistical tests to be used
- The greater the sample size, the more confident we can be that any *sample statistic* represents the 'population' from which the sample is drawn

Confidence intervals

- Confidence intervals: from the sample result, the range of values within which we can be x% confident that the true 'population' statistic actually lies
- 95% confidence interval: within this range of the sample statistic, we can be 95% confident that the population statistic actually lies

Distribution of sample means



Z Statistic

- We can still use our Z statistics and the Z tables since the *distribution of sample means* is normal

- $$Z = \frac{\overline{X}_i - \mu}{\sigma / \sqrt{n}}$$ or $$Z = \frac{\overline{X}_i - \mu}{s / \sqrt{n}}$$

Question

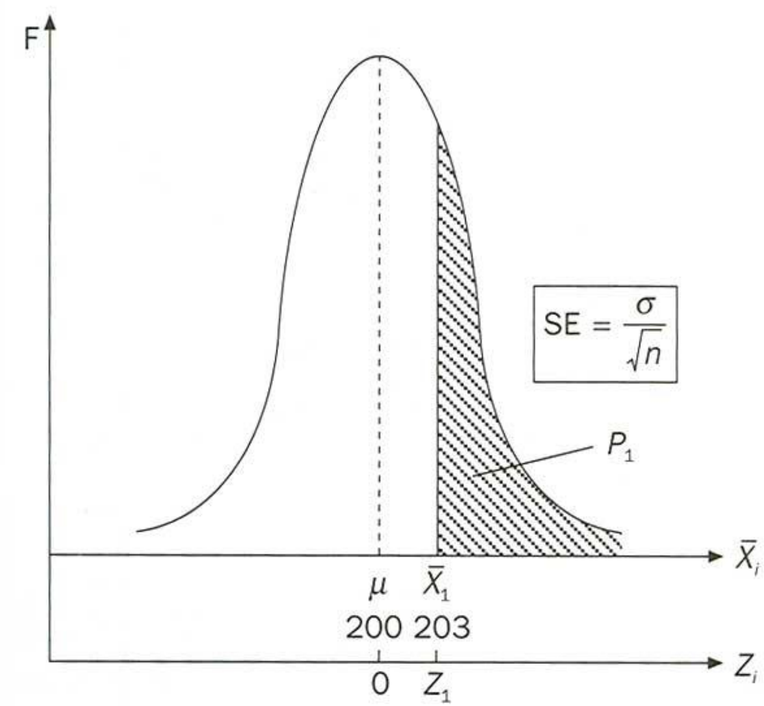
- A production line makes unit which are normally distributed with a mean weight of 200 grams and a standard deviation of 9 grams. What is the probability of a sample of 36 units having a (sample) mean weight of 203 grams or more?

$$Z_1 = \frac{203 - 200}{9/\sqrt{36}} = \frac{3}{1.5}$$

$Z_1 = +2$

$P_1 = 0.0228$

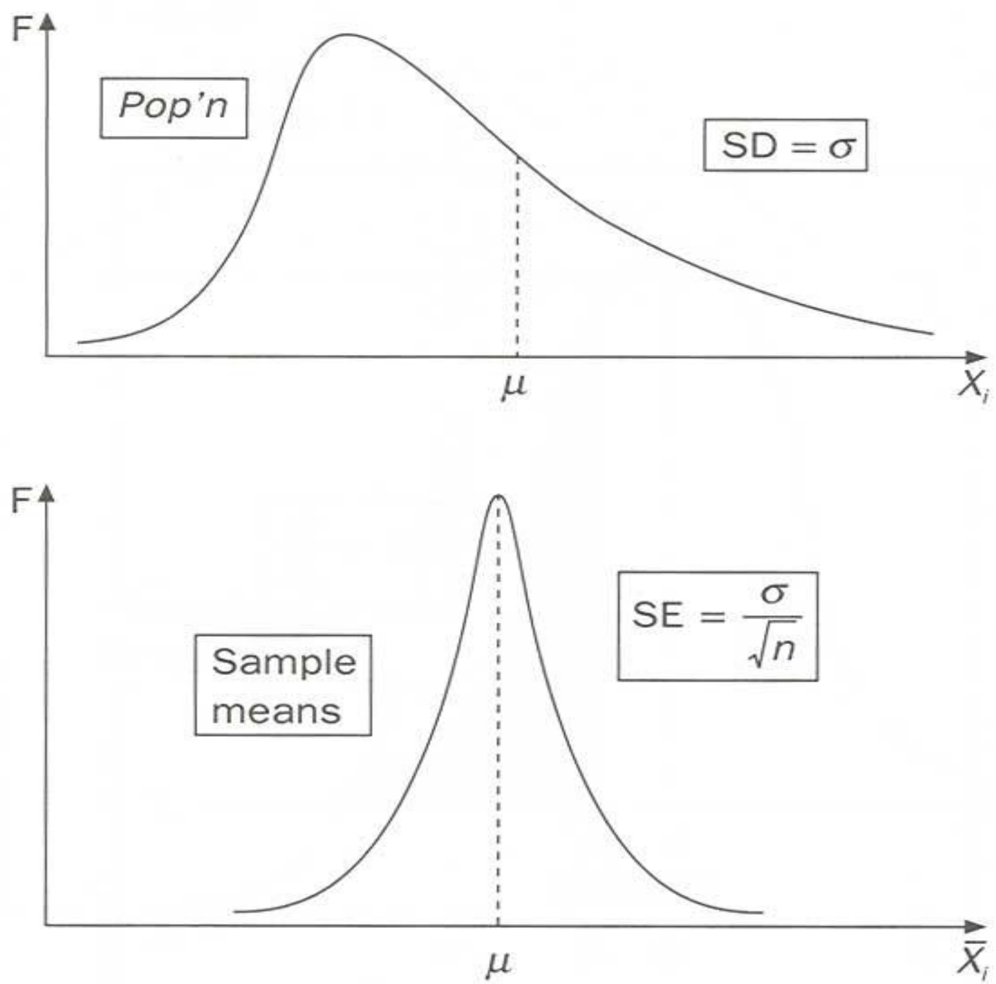
Solution $P_1 = 0.0228$



Central Limit Theorem

- Even if the population is not normal, if sampling is random and the sample size (n) is $\geq$ 32, then the *distribution of sample means* can be regarded as approximately normal
- We can then still use our Z statistic and Z tables for calculating probabilities for this distribution of sample means

Central Limit Theorem



Question

- The output of glass panels has a mean thickness of 4cm and a standard deviation of 1cm. If a random sample of 100 glass panels is taken, what is the probability of the sample mean having a thickness of between 3.9cm and 4.2cm?

$$Z_1 = \frac{3.9 - 4.0}{1/\sqrt{100}} = \frac{-0.1}{0.1} = -1$$

$P_1 = 0.1587$

$A_1 = 0.5000 - 0.1587$

$A_1 = 0.3413$

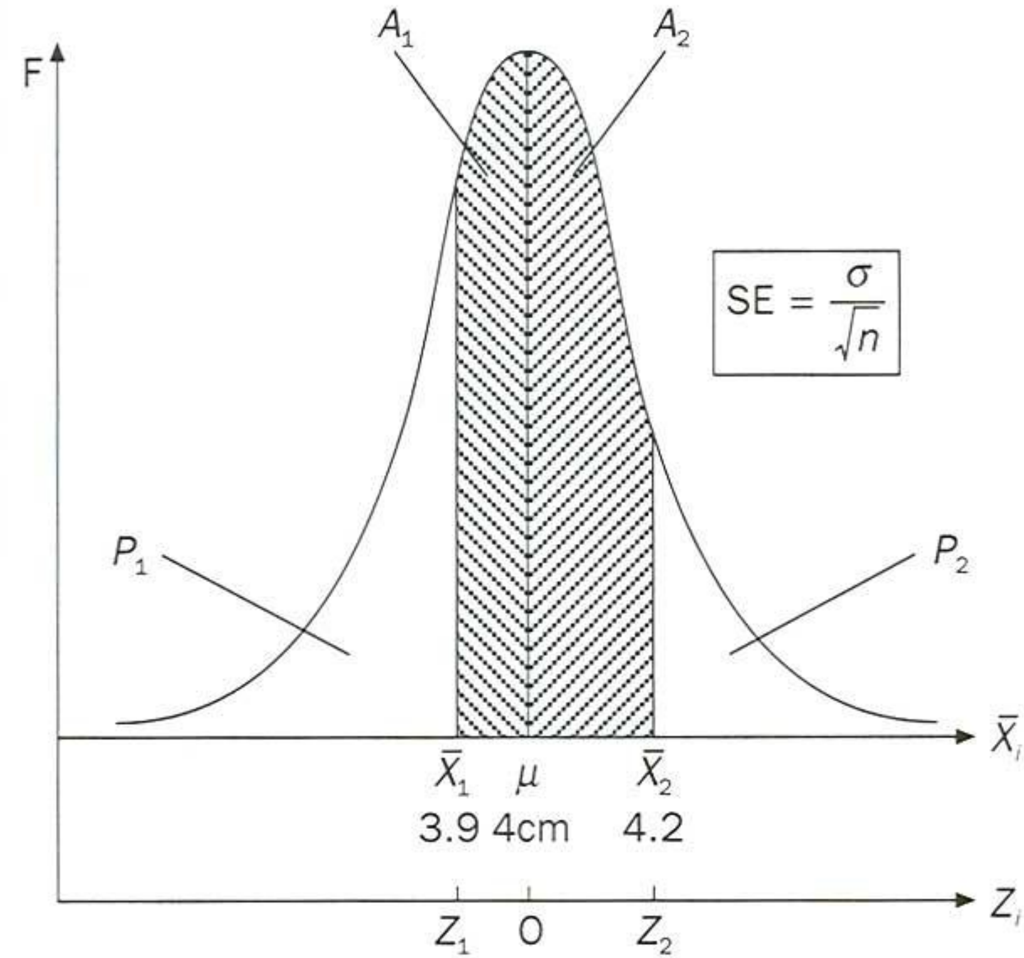
$$Z_2 = \frac{4.2 - 4.0}{1/\sqrt{100}} = \frac{0.2}{0.1} = +2$$

$P_2 = 0.0228$

$A_2 = 0.5000 - 0.0228$

$A_2 = 0.4772$

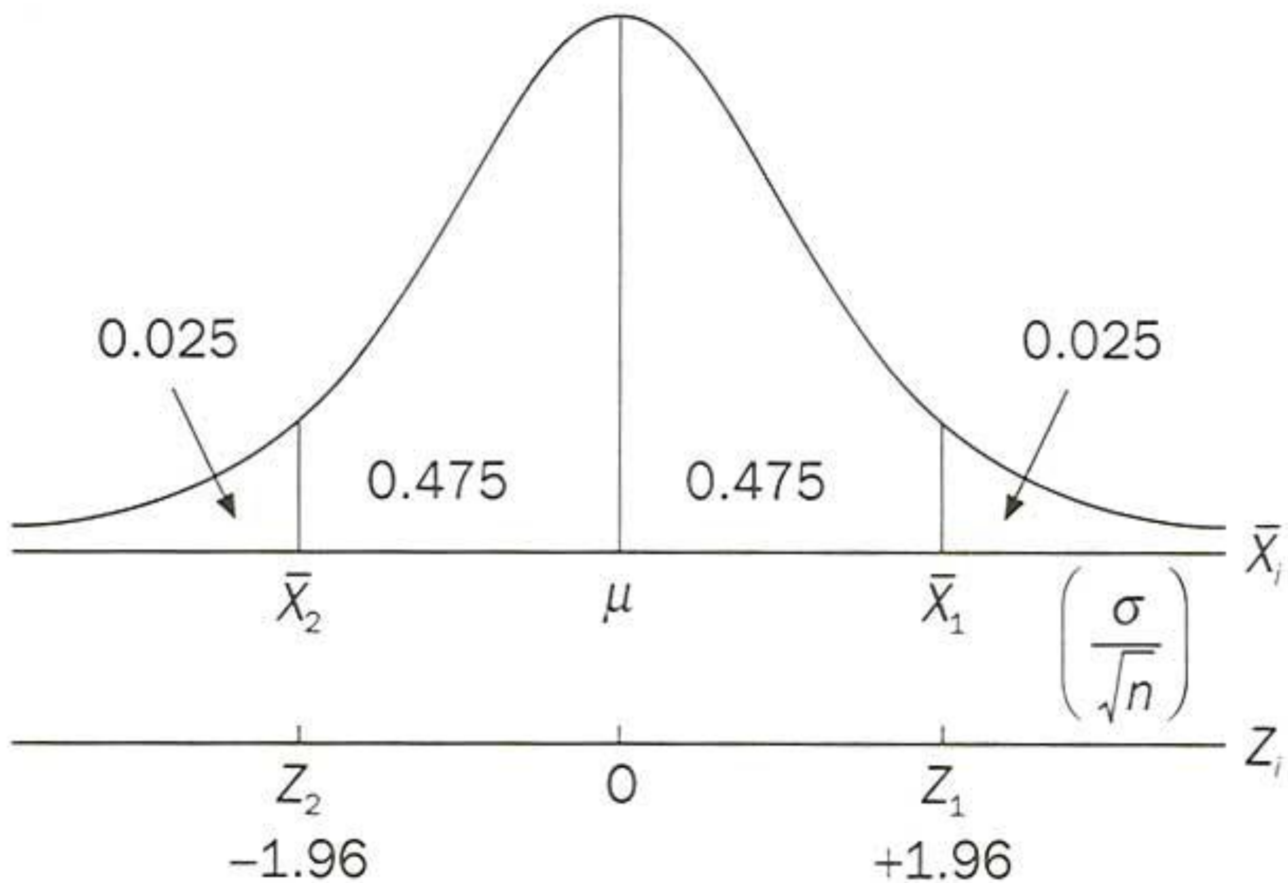
Solution
$A_1 + A_2 = 0.8185$



Confidence intervals

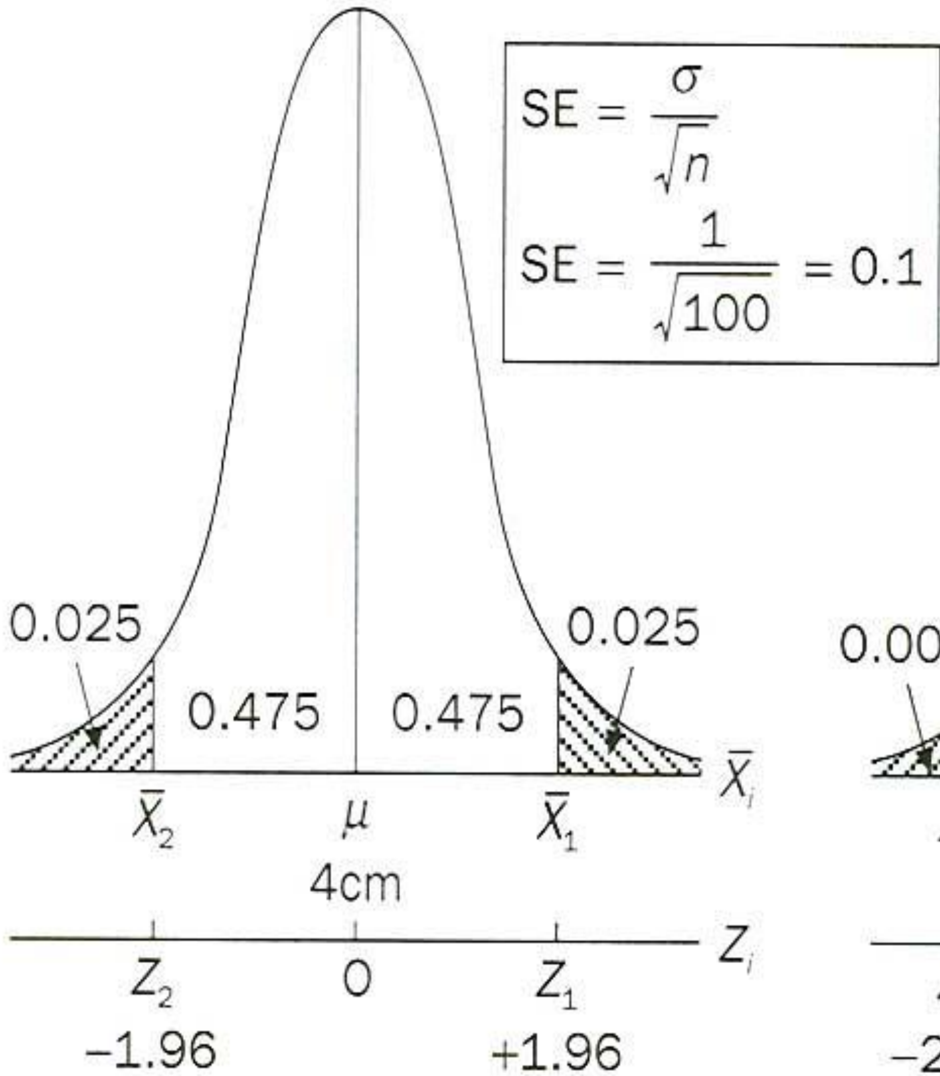
- A confidence interval is a range of values within which we can have a certain level of confidence that a particular value of a variable will lie
- 95% and 99% confidence intervals are the most usual

95% confidence interval for sample mean

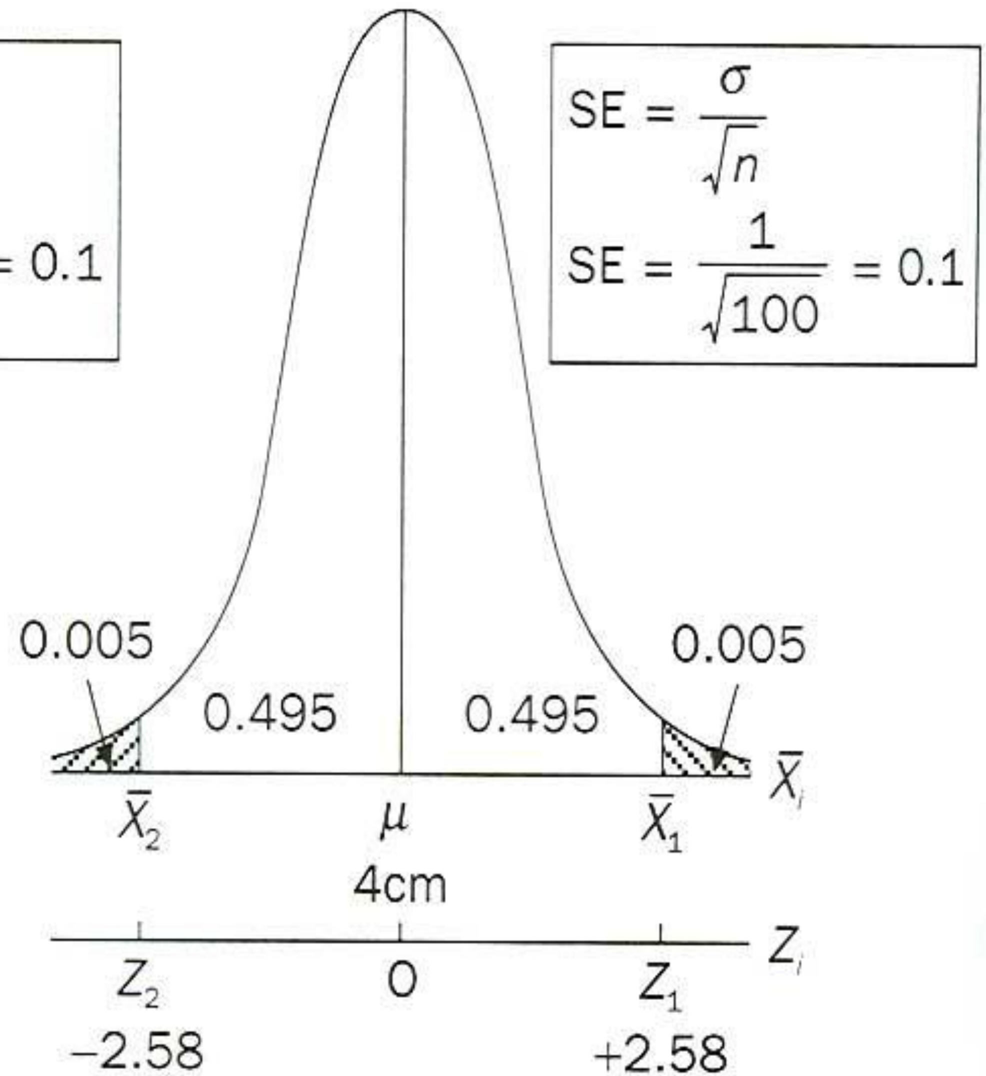


Question

- A large number of random samples of size 100 are taken from the production line of glass panels which is thought to produce panels with mean thickness 4cm and standard deviation 1cm
- Find a) the 95% and b) the 99% confidence intervals for the sample mean



(a) Finding the 95% confidence interval for the sample mean



(b) Finding the 99% confidence interval for the sample mean

Solutions

a)

$$4 \pm 1.96\ 1/\sqrt{100}$$

$$4 \pm 0.196$$

3.804 to 4.196 cm

b)

$$4 \pm 2.58\ 1/\sqrt{100}$$

$$4 \pm 0.258$$

3.742 to 4.258 cm